TRINITY COLLEGE

YEAR 11
MATHEMATICS
SPECIALIST

Test 3, 2023
Section One: Calculator Free
Trigonometry & Matrices

STUDENT'S NAME: MARKING KEY [KRISZYK]

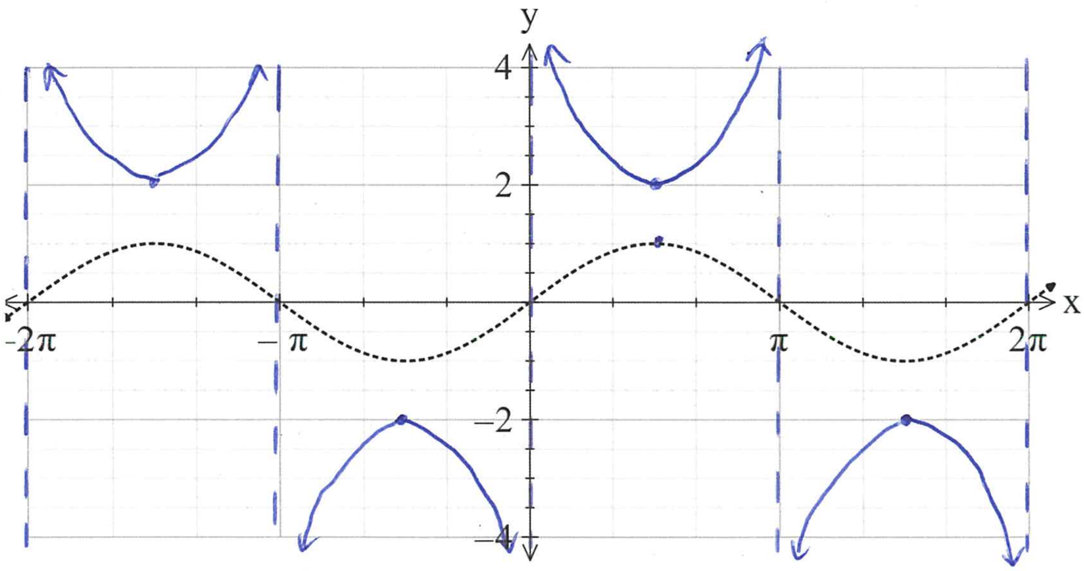
DATE: Monday 7th August **TIME:** 35 minutes **MARKS:** 38
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 1 (6 marks)

(a) The graph of $y = \sin(x)$ is shown below. On the same set of axes draw the graph $y = 2\csc(x)$. (3 marks)



asym ✓
T.P ✓
shape ✓

(b) State the amplitude, period and the coordinates of the y-intercept for $y = 3\sec\left(\frac{x}{2}\right)$

(i) Amplitude: N/A ✓ (1 mark)

(ii) Period: 4π ✓ (1 mark)

(iii) Coordinates of y-intercept: $(0,3)$ ✓ (1 mark)

Question 2 **(8 marks)**

(a) Consider the following matrices.

$A = \begin{bmatrix} 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 2 & 4 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 2 \\ 5 & 4 \end{bmatrix}$

Using these matrices, determine matrices P, Q and R below. If it is not possible to calculate a matrix, explain why.

(i) $P = CD$ (2 marks)

$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 20 \\ 3 & 8 \end{bmatrix}$

(ii) $Q = AD + AC$ (3 marks)

$Q = A(D+C)$

$D+C = \begin{bmatrix} -1 & 2 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 7 & 5 \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 48 & 40 \end{bmatrix}$

(b) Given $E = \begin{bmatrix} x & 1 \\ x-1 & 1 \end{bmatrix}$

(i) Calculate det(E). (1 mark)

$ad - bc$

$= (x)(1) - (1)(x-1)$

$= x - x + 1$

$\det(B) = 1$

(ii) Show that there exists a value of x such that $E^{-1} = -E$ (2 marks)

$-B = \begin{bmatrix} -x & -1 \\ -x+1 & -1 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 1 & -1 \\ -x+1 & x \end{bmatrix}$

$\begin{bmatrix} -x & -1 \\ -x+1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -x+1 & x \end{bmatrix}$ $\therefore x = -1$

Question 3 **(7 marks)**

(a) Prove that $cos2\theta = 2cos^2\theta - 1$ (3 marks)

$$\cos 2\theta = \cos(\theta + \theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1$$

(b) Solve $3\csc 2\theta = -2\sqrt{3}$ over the domain $\overset{-\pi}{\cancel{0}} \le \theta \le \pi$ (4 marks)

$$\csc 2\theta = -\frac{2\sqrt{3}}{3}$$

$$\frac{1}{\sin 2\theta} = \frac{3}{-2\sqrt{3}}$$

$$\sin 2\theta = -\frac{3}{\sqrt{2}}$$

$$\theta = \frac{-\pi}{3}, \frac{-\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

Question 4 **(5 marks)**

Prove $1+2\cos 2A+\cos 4A=8\cos^4 A-4\cos^2 A$.

Hint: $\cos 4A=\cos(2(2A))$

$$\begin{aligned}
LHS &= 1+2\cos 2A+\cos 4A \\
&= \cancel{1}+2\cos 2A+(2\cos^2 2A-\cancel{1}) \\
&= 2\cos 2A+2\cos^2 2A \\
&= 2\cos 2A(1+\cos 2A) \\
&= 2(2\cos^2 A-1)(\cancel{1}+2\cos^2 A-\cancel{1}) \\
&= 4\cos^2 A(2\cos^2 A-1) \\
&= 8\cos^4 A-4\cos^2 A \\
&= RHS
\end{aligned}$$

Q.E.D.

Question 5 **(4 marks)**

Prove $\tan\theta + \cot\theta = \dfrac{2}{\sin 2\theta}$

$$LHS = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \quad \checkmark$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \quad \checkmark$$

$$= \frac{1}{\sin\theta\cos\theta} \quad \checkmark$$

$$= \frac{1}{\frac{1}{2}\sin(2x)} \quad \checkmark$$

$$= \frac{2}{\sin 2x}$$

$$= RHS \quad Q.E.D$$

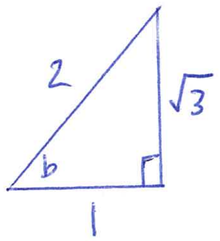
Question 6 **(8 marks)**

(a) Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $a\sin(\theta + b)$ (4 marks)

$a\sin(\theta+b) = a[\sin\theta\cos b + \cos\theta\sin b]$

$\therefore \cos b = 1$

$\sin b = \sqrt{3}$



$R = \sqrt{1^2 + \sqrt{3}^2}$

$R = 2$

$a = 1$

$b = \sqrt{3}$

$\tan \frac{b}{a} = \tan\sqrt{3}$

$\alpha = \frac{\pi}{3}$

$= 2\sin\left(\theta + \frac{\pi}{3}\right)$

(b) Evaluate $\cos 15^\circ - \cos 105^\circ$ as an exact value. (4 marks)

$\cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

$= -2\sin 60 \sin(-45)$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{\sqrt{2}}$ or $\frac{\sqrt{6}}{2}$

END OF QUESTIONS

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Test 3, 2023
Section Two: Calculator Allowed
Trigonometry & Matrices

STUDENT'S NAME: MARKING KEY [KRISZYK]

DATE: Monday 7th August **TIME:** 15 minutes **MARKS:** 12
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 7 **(3 marks)**

Determine all the values of k for which the matrix $\mathbf{M}^{-1}$ exists, where:

$$\mathbf{M} = \begin{bmatrix} -4 & -2 \\ 3 & 1 \end{bmatrix} + k\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -4+2k & -2+k \\ 3-k & 1 \end{bmatrix} \checkmark$$

For inverse to exist $(-4+2k)(1) - (3-k)(-2+k) \neq 0$ ✓

Solve $k \neq 1, \ k \neq 2$ ✓

Question 8 **(8 marks)**

(a) Let matrix $A = \begin{bmatrix} 2 & -2 \\ 7 & -6 \end{bmatrix}$

(i) Determine A^{-1}. (1 mark)

$\frac{1}{2}\begin{bmatrix} -6 & 2 \\ -7 & 2 \end{bmatrix}$ ✓

(ii) Express the equations $7a - 6b = 23$ and $2a - 2b = 7$ as a matrix. (1 mark)

$\begin{bmatrix} 2 & -2 \\ 7 & -6 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 23 \end{bmatrix}$ ✓

(iii) By using your answer from part (i) use matrix algebra to solve the equations in part (ii). (2 marks)

$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -6 & 2 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 23 \end{bmatrix}$ ✓

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1.5 \end{bmatrix}$ ✓

(b) Solve the equation $\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} B = B + \begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$ for the 2×2 matrix B (4 marks)

let $X = \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix}$ $Y = \begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$

$XB = B + Y$

$XB - B = Y$

$(X - I)B = Y$ ✓

$B = (X-I)^{-1}Y$ ✓

$B = \begin{bmatrix} 2 & 0 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$ ✓

$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ ✓

END OF QUESTIONS